Synopsis for Ph.D.

TOPIC : "SOME STUDIES IN FIXED POINT THEORY FOR DIFFERENT SPACES"

1. Introduction

In recent years much interest has been Shown by various researchers/authors towards investigations in fixed point theory in different spaces, In fact the theory of fixed point is one of the powerful tools of modern mathematics. If is not only used on daily basis in pure and applied mathematics but it is also useful to act as a bridge between analysis and topology and provides a a very nice area of interaction between two. Further fixed point theory yields Conditions under which maps have solutions. Today the theory of fixed point has emerged as a very powerful and important tool in the investigation of non-linear phenomenon. In particular, fixed point techniques are applicable in such diverse fields like Biology [1(a)], chemistry, Economics [2], Engineering, Game Theory [2] and also in some branches of Social Science[1] as well as to some extent in Computer Science

As a matter of fact, fixed point theorems deal with the existence of fixed point.

Below we give some relevant definitions and theorems which are useful in the building of our research project/thesis.

Definition 1:

Let (X,d) be a metric space with metric d. Let T:X→X. Let $x_0 \epsilon X$ then x_0 is Called a fixed point of the mapping T. There are mappings, which have no fixed points and also there are mapping, which have fixed points and sometimes non-unique. During the last 40 years research workers have devoted to the theory of operators in order to find the fixed point of the operators.

Edelstein [4] introduced the Concept of locally Contractiveness and extended Banach's Contraction mapping theorem which runs as follows.

Theorem1: Let X be a Complete metric space and let T be an operator mapping from X into itself such that

$d(T_x, T_y) \leq \alpha\, d(x,y)$ -.....(1.1)

Where $0 \leq \alpha < 1$ and for all $x,y \epsilon X$.

Then, their exists a unique fixed point of the operator T in X

Mapping satisfying (1.1) are known as Contraction mapping

Definition 2:

A mapping T from X into itself is said to be locally Contractive if for every $x \in X$ there exists ϵ and λ ($\epsilon > 0, 0 \leq \lambda < 1$) which may depend on X.

Such that p, q $\in S(x, \epsilon)$ = {y:d(x,y) < ∈ }implies $d(T_p, T_q) < \lambda d(p,q)$

Kannan [6,7] gave a different direction to the investigation for existence of fixed point operators Satisfying some Conditions on the operators. Ciric [3] introduced the Concept of quasi-Contraction mapping.

Definition 3 :

Let (X,d) be a metric space and T:$X \to X$ be a mapping such that there exist $\varepsilon > 0$ for which $d(T_p, T_q) < (p,q)$ whenever $d(P,q) < \varepsilon$ and P≠q.

Then T is Called a Contractive [Edelstein, 1962] Edelstein [4] proved that

if $\lim_{i\to\infty} T^{n_i} x = x_0 \in X$ for Some $x \in X$ then an ∈ Contractive mapping T has x_0 as a periodic point, i.e $T_{x_0}^k = x_0$ for Some positive integer K>1. The point x_0 turns out to be fixed point if $d(T_{x_0}, x_0) < \in$ is Considered (Edelstein [4]) The concept of ∈- Contractiveness has been extended to well-locally Contractiveness by Ciric [3] Who proved the existence of a periodic point for such mapping in a metric Space. Tiwary and Lahiri [11] generalised these definitions and proved few fixed point theorems for some other workers in the field see ref.[5,8-10,12-14]

Definition 4:

Let B(x) be the family of all bounded subsets of X. Let F:X⟶B(x). Then F is Called a set valued mappings. Nadler [15], Fisher [16] ete have obtained Several fixed point theorems for set valued mappings.

Definition 5:

Two Self mapping A and S of a metric space (X,d) are said to be compatible if $\lim_{n\to\infty} d(ASx_n, SAx_n) = 0$ whenever $\{x_n\}$ is a sequence X Such that $\lim_{h\to\infty} A x_n = \lim_{h\to\infty} S_{x_n} = y$ for some y in X.

Definition 6:

Czerwik [17]: Let X be a non-empty set and $d: X \times X \to [0, \infty]$. A function d is called a b-metric with Constant (base) s≥ 1if

i) d(x,y) =0 iff x=y

(ii) d(x,y) $= d(y,x) = for\ all\ x, y \in X.$

(iii) $d(x,y) \leq s\,[d(x,z) + d(z,y)]\ for\ all\ x, y, z \in X.$

In this case the pair (X, d) is called a b-metric space [X_{ie} *et al* [18] It is ovious that a b-metric Space with s=1 in a metric space.

Definition 7 :

Let X be a non-empty set and let D(X) be a Collection of Subsets of $X \times X$, then D(X) is Called the uniformity for X when it satisfies.

i) $D \in D(X) \Rightarrow D \subset D$

ii) $D_1, D_2 \in D \Rightarrow D_1 \cap D_2 \in D$

iii) $D \in D \Rightarrow EoE \subset D \text{ for some } E \subset D$

iv) $D \in D \Rightarrow E^{-1} \subset D \text{ for some } E \in D$

v) $D \in D, D \subset E \Rightarrow E \subset D$

when X has such a structure, we Call X is a uniform space.

We shall use above definitions, theorems and other related and relevant matters of functional analysis in preparation of our thesis as and when required.

2. Hypothesis:

The proposed work will be about the existence of fixed points in different spaces and Some new type of metric Spaces by using some Contractive mappings, set valued mappings in Banach spaces alongwith Common fixed point theorems for Compatible mappinges.

3. Methodology:

In our proposed work we shall use different relevant definitions, propositions, theorems and Corollaries established by earlier researchers using different journals and reference books and networking .

4. Purpose and Need of Study :

Here in this work, our purpose is to provide new results and theorems in fixed point theory for different spaces by introducing Contractive type mappings, set valued mappings and Compatible mappings. Such type of study in functional analysis is much needed for wider and deeper knowledge in functional analysis Specially in fixed point theory. Further our Study is very useful in Game theory, Economics, Biology, Social Science and Computer Sciences.

5. Work plan (Chapterization): -

Our thesis entitled **"Some studies in Fixed Point Theory for Different spaces"** will be divided into the following six chapters. Each Chapter will be divided into sections and equations/theorems will be numbered section wise. A good number of relevant references will be given at the end of the thesis.

The Chapters are:

1. Introduction (with literature review)
2. Some fixed point theorems for Contractive type mappings in metric spaces.
3. Fixed point theorems for set-valued mappings in Banach spaces.
4. Fixed point theorems for multi-valued mappings in uniforms space.
5. Some Common fixed point theorems for multi- valued mappings in b-metric Spaces.
6. Some Common fixed point theorems for Compitable mappings.

Changes in some Chapters will be done if necessarily needed while preparation of the thesis/Project.

6. References:

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